THE LENGTH OF A DEGREE OF LATITUDE AND LONGITUDE FOR ANY PLACE.

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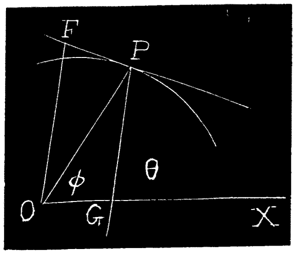
Let O be the center of the earth, P any place on the surface, PG the normal and PF the tangent at P, $OP=p$=the perpendicular from the center on the tangent. a, b the semi-axes of the earth, ρ=radius of curvature at P, $OP=r$ =the radius vector, $\angle POX=\varphi$, $\angle PGX=FOX=\theta$, l=length of a degree of meridian, ρ'=radius of circle of latitude, L=length of degree of latitude, $e^2=\dfrac{a^2-b^2}{a^2}=00680349$=square of the eccentricity, since a=6378190 meters=3963.296 miles.



$$\text{Now } \rho=\frac{a^2b^2}{p^3},\ \text{but } p=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}=a\sqrt{1-e^2\sin^2\theta}.$$

$$\therefore\ \rho=\frac{a(1-e^2)}{(1-e^2\sin^2\theta)^{\frac{3}{2}}},\quad l=\frac{2\pi\rho}{360}=\frac{\pi\rho}{180}.$$

$$\therefore\ l=\frac{\pi a(1-e^2)}{180(1-e^2\sin^2\theta)^{\frac{3}{2}}}=\frac{68.70175}{(1-e^2\sin^2\theta)^{\frac{3}{2}}}\text{ miles}=\frac{110562.7346}{(1-e^2\sin^2\theta)^{\frac{3}{2}}}\text{ meters.}$$

$$\frac{1}{(1-e^2\sin^2\theta)^{\frac{3}{2}}}=1+\tfrac{3}{2}e^2\sin^2\theta+\tfrac{15}{8}e^4\sin^4\theta+\tfrac{105}{48}e^6\sin^6\theta+\ldots.$$

The fourth term can be omitted as its greatest value will not affect the result more than three inches.

$$\sin^2\theta=\tfrac{1}{2}(1-\cos2\theta),\quad \sin^4\theta=\tfrac{1}{8}(3-4\cos2\theta+\cos4\theta).$$

These values, with the value of e, give

$$\frac{1}{(1-e^2\sin^2\theta)^{\frac{3}{2}}}=1.00514058-.00515323\cos2\theta+.00001265\cos4\theta.$$

$$\therefore\ l=69.054917-.354036\cos2\theta+.000869\cos4\theta\text{ miles}$$

$$=111131.09118-569.75520\cos2\theta+1.39862\cos4\theta\ \text{ meters.}$$

$$\rho'=r\cos\varphi=\frac{a\cos\theta}{\sqrt{(1-e^2\sin^2\theta)}}.$$

$$L=\frac{2\pi\rho'}{360}=\frac{\pi\rho'}{180}=\frac{\pi a\cos\theta}{180\sqrt{(1-e^2\sin^2\theta)}}.$$

$$\therefore\ L=\frac{69.1726\cos\theta}{\sqrt{(1-e^2\sin^2\theta)}}\text{ miles}=\frac{111320.4635\cos\theta}{\sqrt{(1-e^2\sin^2\theta)}}\text{ meters.}$$

$$\frac{1}{\sqrt{(1-e^2\sin^2\theta)}}=1+\tfrac{1}{2}e^2\sin^2\theta+\tfrac{3}{8}e^4\sin^4\theta+\tfrac{15}{48}e^6\sin^6\theta+\ldots.$$

$$=(1+\tfrac{1}{2}e^2+\tfrac{3}{8}e^4)-(\tfrac{1}{2}e^2+\tfrac{3}{4}e^4)\cos^2\theta+\tfrac{3}{8}e^4\cos^4\theta\ldots.$$

$$\cos^3\theta=\tfrac{1}{4}(\cos3\theta+3\cos\theta),\quad \cos^5\theta=\tfrac{1}{16}(\cos5\theta+5\cos3\theta+10\cos\theta).$$

$$\therefore\ L=111320.4635[(1+\tfrac{1}{8}e^2+\tfrac{3}{64}e^4)\cos\theta-(\tfrac{1}{8}e^2+\tfrac{9}{128}e^4)\cos3\theta+\tfrac{2}{128}e^4\cos5\theta]$$

$$=111415.37533\cos\theta-95\ 03428\cos3\theta+.12022\cos5\theta.$$

Since the greatest value of the last term is not over five inches it can be omitted.

$$\therefore\ L=111415.37533\cos\theta-95.03428\cos3\theta\text{ meters}$$

$$=69.23155\cos\theta-.05905\cos3\theta\text{ miles.}$$

The following table gives the length of a degree at intervals of five degrees.

	l		L	
Degrees	*Meters*	*Miles*	*Meters*	*Miles*
0°	110562.7346	68.70175	111320:3411	69.17250
5°	110571.305	68.70701	111051.725	69.00558
10°	110596.769	68.72288	109640.673	68.12878
15°	110638.365	68.74873	107552.254	66.83108
20°	110694.879	69.78386	104648.397	65.02667
25°	110764.615	68.82719	100952.272	62-72996
30°	110845.514	68.87746	96489.058	59.95660
35°	110935.152	68.93315	91290.501	56.72631
40°	111030.839	68.99261	85396.151	53.06366
45°	111129.693	69.05405	78850.126	48.99608
50°	111228.715	69.11556	71698.992	44.55249
55°	111324.887	69.17532	63997.427	39.96687
60°	111415.270	69.23150	55802.722	34.67483
65°	111497.081	69.28234	47178.162	29.31568
70°	111567.789	69.32627	38188.589	23.72972
75°	111625.216	69.36194	28903.727	17.96027
80°	111667.556	69.38825	19394.797	12.05159
85°	111693.506	69.40438	9735.561	6.04951
90°	111702.245	69.40988	0.000	0.000